Province of the
EASTERN CAPE
EDUCATION

NATIONAL
SENIOR CERTIFICATE

GRADE 12

JUNE 2017

MATHEMATICS P1

MARKS: **150**

TIME: **3 hours**

MATHE1

This question paper consists of 11 pages, including an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non- graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An information sheet, with formulae, is included at the end of the question paper.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x, in each of the following:

1.1.1 $x^2 - x - 30 = 0$ (3)

1.1.2 $3x^2 + x - 1 = 0$ (correct to TWO decimal places) (3)

1.1.3 $x^2 \leq 2(x + 4)$ (4)

1.1.4 $3x - 5\sqrt{x} = 2$ (5)

1.2 Solve simultaneously for x and y in the following equations:

$y - x - 6 = 0$ and $(x - 3)^2 + (y - 3)^2 = 18$ (5)

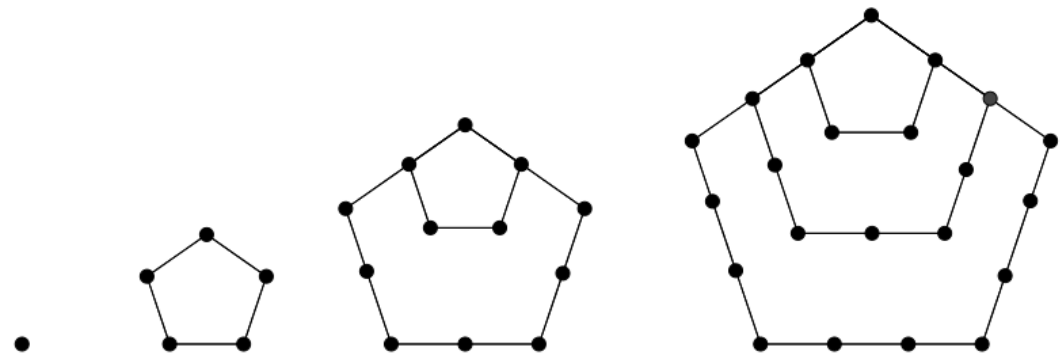
1.3 Solve for x if: $1 + \dfrac{1}{x + \dfrac{1}{x}} = \dfrac{7}{5}$; $x \neq 0$

(5)

[25]

QUESTION 2

2.1 A pentagon number can be represented by dots that are arranged in the shape of a pentagon, as shown below. The first four pentagonal numbers are given.



$T_1 = 1$ $T_2 = 5$ $T_3 = 12$ $T_4 = 22$

2.1.1 Determine the next two pentagon numbers. (2)

2.1.2 Determine an expression for the n^{th} term of the sequence. (4)

2.1.3 Determine which term in the pentagon number pattern will be equal to 3432. (4)

2.2 If $-\sqrt{2}\ ;\ m\ ;\ 3\sqrt{2}$ are the first three terms of an arithmetic sequence:

2.2.1 Determine the value of m (2)

2.2.2 Determine T_{51} (leave your answer in surd form) (3)

2.3 How many terms between 50 and 500 are divisible by 7? (3)

2.4 Given the geometric sequence: $2\ ;\ \frac{2}{3}\ ;\ \frac{2}{9}\ ;\ \ldots$

2.4.1 Determine the general term for the sequence in the form $a.b^n$ (3)

2.4.2 Is the geometric sequence convergent? Give a reason for your answer. (2)

2.4.3 Solve for p, if $3^p = S_\infty - S_4$ (5)

2.5 Expand and evaluate: $\sum_{k=1}^{6}\left(\sum_{n=1}^{k} 1\right)$ (2)

[30]

QUESTION 3

3.1 Given a function, $f:\ y + 4 = (x-5)^2$

3.1.1 Write down the equation of the axis of symmetry of f. (1)

3.1.2 Determine the x-intercepts of f. (3)

3.1.3 Sketch the graph of f, clearly showing the intercepts with the axes and the turning point. (4)

3.1.4 Write down the range of f. (1)

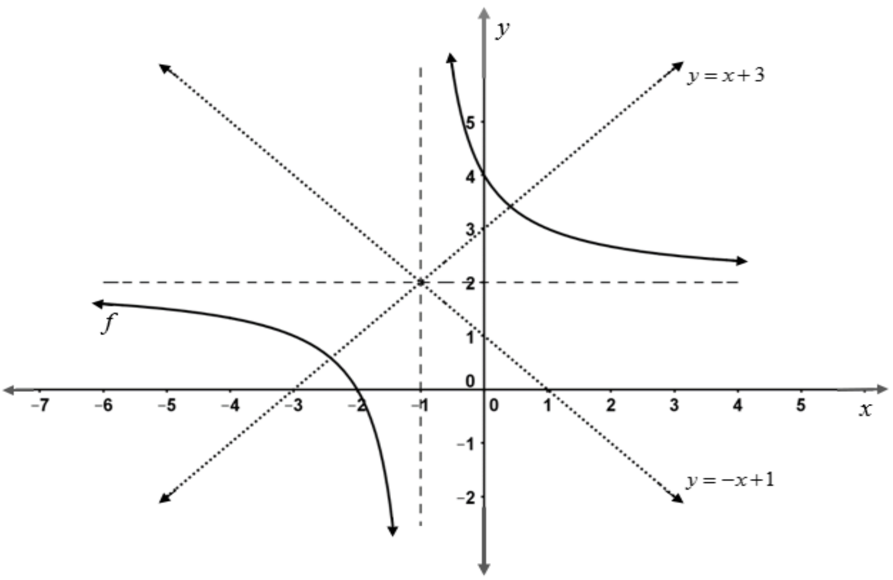
3.1.5 $f(x)$ is transformed to $g(x)$, where the x-intercepts of $g(x)$ is the same as that of $f(x)$ and the turning point of $g(x)$ is $(5\ ;\ 4)$.
Describe the transformation and write down the equation of $g(x)$. (2)

3.2 Given: $f(x) = x^2 + 3$ and $g(x) = kx - 1$, determine the value(s) of k if g a tangent to the graph of f. (5)

[16]

QUESTION 4

Given the graph of f, a hyperbola of the form $y = \dfrac{a}{x+p} + q$, answer the questions that follow.



4.1 Write down the values of p and q. (2)

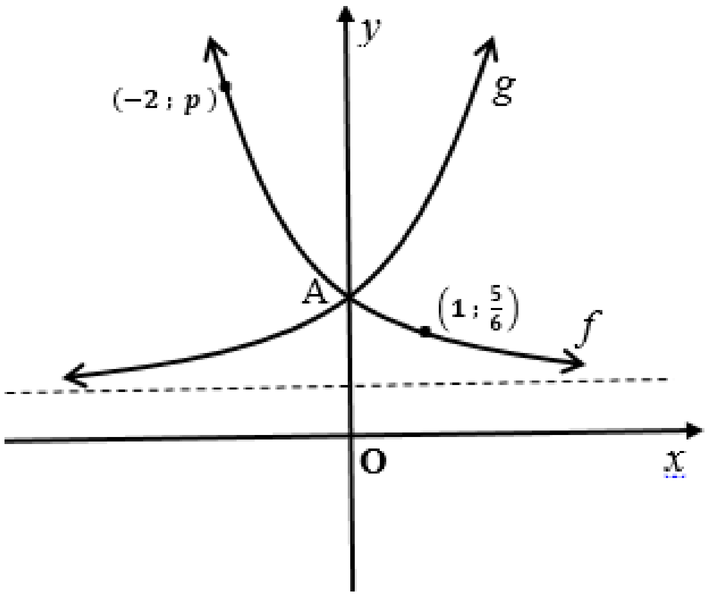
4.2 Determine the value of a, and write down the equation of f in the form $y = \ldots$ (3)

4.3 The axes of symmetry of f are $y = x + 3$ and $y = -x + 1$. The graph of f is transformed to g such that the axes of symmetry of g are given by $y = x - 3$ and $y = -x + 1$. Describe the transformation. Show all calculations to support your answer. (5)

[10]

QUESTION 5

The graph of f defined by $f(x) = a^x + \frac{1}{2}$, where $a > 0$ and $a \neq 1$, passes through the points $(-2\,;\,p)$ and $\left(1\,;\,\frac{5}{6}\right)$, is drawn in the sketch below with the graph of g .



Use the sketch and the given information to answer the following questions.

5.1 Determine the value of a. (2)

5.2 Find the value of p . (2)

5.3 Write down an equation for g , the reflection of f in the y-axis. (1)

5.4 If $h(x) = g(x) - \frac{1}{2}$, write down the equation of h^{-1} in the form $y = \ldots$ (2)

5.5 Calculate the average gradient of the curve of f between $x = -2$ and point A. (3)

[10]

QUESTION 6

6.1 Calculate the effective interest rate per annum if the nominal interest rate is 15% compounded monthly. (3)

6.2 Neymar applied for a loan of R75 000 from XYZ Bank at 12% simple interest per annum for 8 years.

6.2.1 Calculate Neymar's monthly instalment. (3)

6.2.2 The bank wants to change the interest to a compound interest per annum without affecting Neymar's monthly payment. Calculate the compound interest rate that the bank would charge correct to 3 decimal places. (4)

6.3 R60 000 is invested in an account which offers interest at 7% p.a. compounded quarterly for the first 18 months. Thereafter the interest rate changes to 5% p.a. compounded monthly. Three years after the initial investment, R5 000 is withdrawn from the account. How much will be in the account at the end of 5 years? (4)

[14]

QUESTION 7

7.1 Determine the derivative of $f(x) = -2x^2$ from first principles. (4)

7.2 Determine:

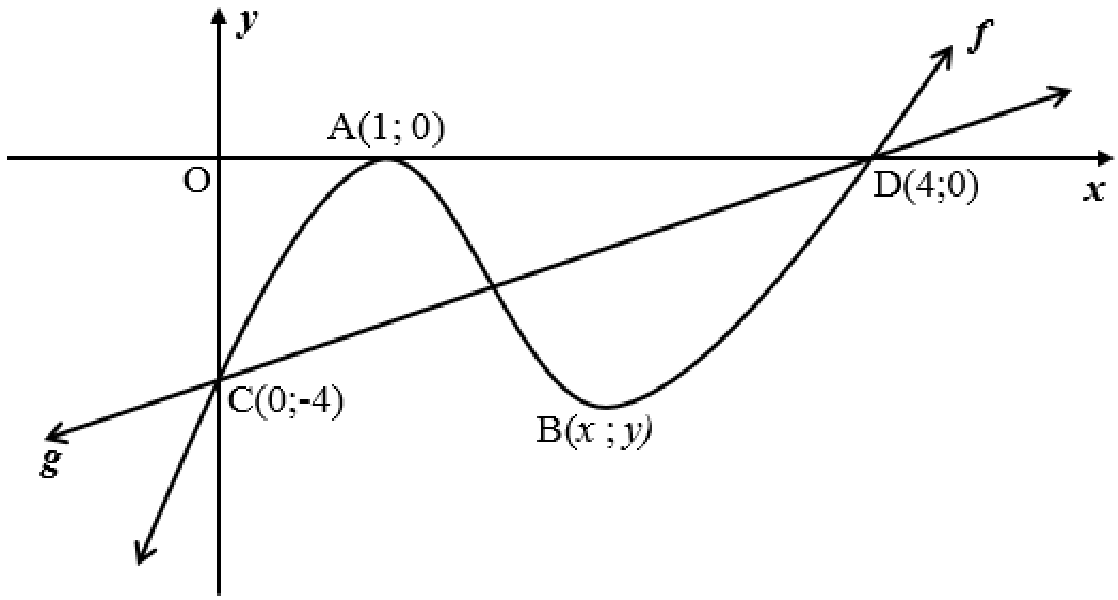
7.2.1 $\frac{dy}{dx}$ if $y = 6x + 4x^2\sqrt{x}$ (3)

7.2.2 $D_t\left[\frac{1-3t^2}{6t^2}\right]$ (3)

[10]

QUESTION 8

The graph of $f(x) = x^3 + bx^2 + cx - 4$ is drawn below. A and B are turning points of f. The x-intercepts and the y-intercept are clearly indicated.



8.1 Show that the values of $b = -6$ and $c = 9$. (3)

8.2 Determine the coordinates of B. (4)

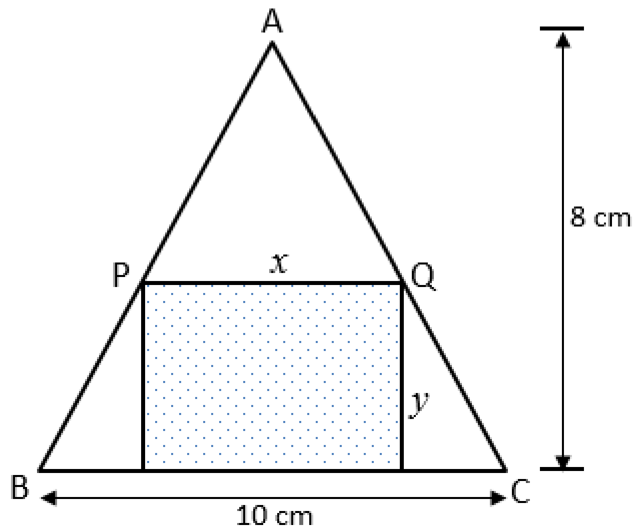
8.3 For which values of x is the graph of f increasing? (2)

8.4 Show that the point of inflection lies on the straight line g that passes through C and D. (4)

[13]

QUESTION 9

A rectangle with length x and width y, is to be inscribed in an isosceles triangle of height 8 cm and base 10 cm, as shown. (Hint: Δ APQ and Δ ABC are similar.)



9.1 Express y in terms of x. (2)

9.2 Hence, show that the area of the rectangle can be express as: $A = 8x - \dfrac{8x^2}{10}$ (2)

9.3 Determine the dimensions, i.e. the length and the width of the rectangle for it to be a maximum. (4)

[8]

QUESTION 10

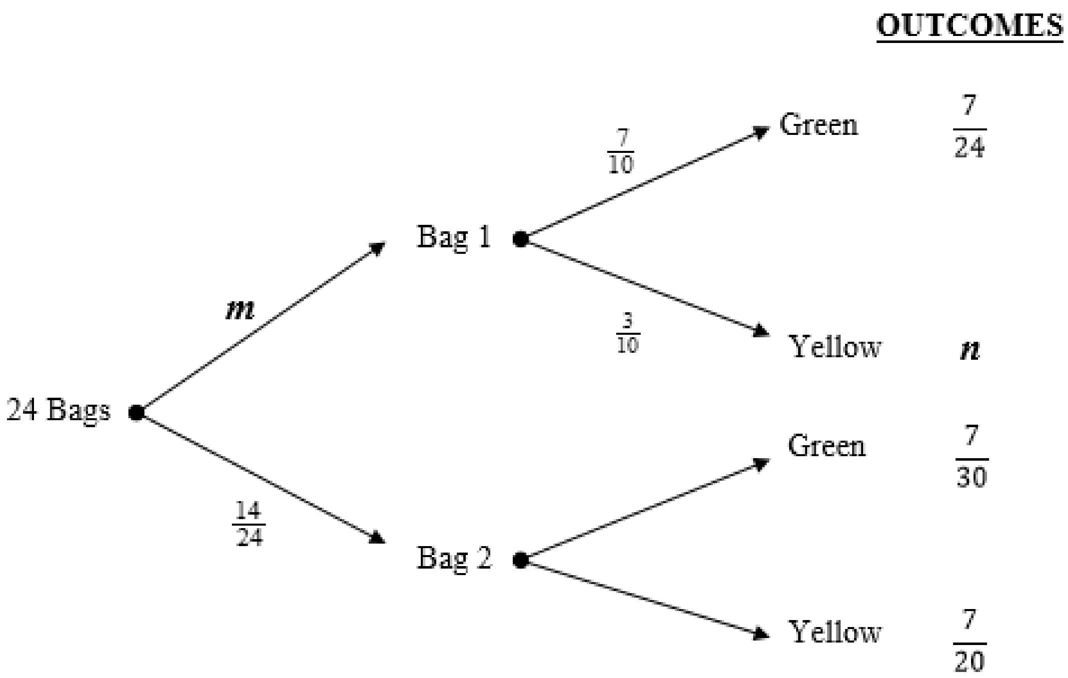
10.1 Let A and B be two events in a sample space. Suppose that the P(A) = 0,4 ; P(B) = k and P(A or B) = 0,7. Determine:

10.1.1 P(A or B)′ (1)

10.1.2 The value of k , for which A and B are mutually exclusive events. (2)

10.1.3 The value of k , for which A and B are independent events. (4)

10.2 There are 24 bags of marbles for sale in a shop. In ten of the bags there are 7 green marbles and three yellow marbles. The other bags each have ***x*** green marbles and 9 yellow marbles. A bag is chosen at random and a marble is then chosen at random from the bag. The tree diagram below illustrates the process and the outcomes.



10.2.1 Determine the values of ***m*** and ***n***. (2)

10.2.2 Determine the value of ***x***. (3)

10.2.3 What is the probability that a marble chosen is green? (2)

[14]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \partial^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$ $\quad P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$